Black holes are formed when a tremendous amount of mass is compressed into an incredibly small volume. Scientists call this tiny region a singularity. Surrounding the singularity is a region of space from which nothing—not even light—can escape once it enters. It is for this reason that black holes are said to be “black.” The outer edge of the region is called the event horizon, and the radius of the event horizon is called the Schwarzschild radius, $R_s$ (Figure 1).

Escape velocity is the minimum velocity needed to break away from an object’s gravitational field. Using the principles of the conservation of energy, the escape velocity formula can be developed and is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

where $M$ is the mass of the object to be escaped from, $R$ is the radius of that object, and $G$ is the gravitational constant.

1. Calculate the escape velocity from Earth.
2. Scientists’ current understanding is that the fastest an object can travel is the speed of light, $c$. By setting $v_{esc} = c$ we can determine the Schwarzschild radius, the radius that a mass must occupy to create an event horizon. Any object compacted below its Schwarzschild radius will become a black hole. Show that when you substitute $v_{esc} = c$ into the equation, the Schwarzschild radius is given by

$$R_s = \frac{2GM}{c^2}$$

3. Suppose we could crush Earth into a volume so small that it became a black hole. Calculate Earth’s Schwarzschild radius. Convert your answer to millimetres.
4. The mass of the supermassive black hole at the center of our Milky Way Galaxy, called Sgr A*, is estimated to be about 4.1 million solar masses. Calculate the Schwarzschild radius of Sgr A*. 1 solar mass = $1.98 \times 10^{30}$ kg.
5. Convert your answer to question 4 to astronomical units. Will any of the stars in the table below be inside the event horizon when they are orbiting at their average radius?
Orbital Data of Six Stars Orbiting the Sgr A*

<table>
<thead>
<tr>
<th>Star</th>
<th>Average Orbital Radius (AU)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>3300</td>
</tr>
<tr>
<td>S2</td>
<td>980</td>
</tr>
<tr>
<td>S8</td>
<td>2630</td>
</tr>
<tr>
<td>S12</td>
<td>2290</td>
</tr>
<tr>
<td>S13</td>
<td>1750</td>
</tr>
<tr>
<td>S14</td>
<td>1800</td>
</tr>
</tbody>
</table>

6. The Andromeda Galaxy is our closest neighbouring galaxy. Scientists have estimated the supermassive black hole at its center to be 100 million solar masses.
   a. Calculate the Schwarzschild radius of this supermassive black hole, and convert your answer to astronomical units.
   b. How many times larger is $R_s$ for Andromeda’s supermassive black hole than $R_s$ for Sgr A*?
1. 
\[ v_{esc} = \sqrt{\frac{2GM}{R}} \]
\[ v_{esc} = \sqrt{\frac{2 \left( 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2 \right) (5.972 \times 10^{24} \text{ kg})}{6.371 \times 10^6 \text{ m}}} \]
\[ v_{esc} = 11200 \text{ m/s} \]

2. 
\[ v_{esc} = \sqrt{\frac{2GM}{R}} \]
\[ c = \sqrt{\frac{2GM}{R_s}} \]
\[ c^2 = \frac{2GM}{R_s} \]
\[ R_sc^2 = 2GM \]
\[ R_s = \frac{2GM}{c^2} \]

3. 
\[ R_s = \frac{2GM}{c^2} \]
\[ R_s = \frac{2 \left( 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2 \right) (5.972 \times 10^{24} \text{ kg})}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} \]
\[ R_s = 0.00885 \text{ m} = 8.85 \text{ mm} \]

4. 
\[ R_s = \frac{2GM}{c^2} \]
\[ R_s = \frac{2 \left( 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2 / \text{kg}^2 \right) (4.1 \times 10^6)(1.98 \times 10^{30} \text{ kg})}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} \]
\[ R_s = 1.20 \times 10^{10} \text{ m} \]

5. 
\[ \frac{1.20 \times 10^{10} \text{ m}}{1} \times \frac{1 \text{ AU}}{1.49 \times 10^{11} \text{ m}} = 0.0808 \text{ AU} \]

Although the average radii are all much larger than 0.0808 AU the minimum radius is not known for any of them.
6a.

\[ R_s = \frac{2GM}{c^2} \]

\[ R_s = \frac{2 \left( 6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2 \right) (100 \times 10^6)(1.98 \times 10^{30} \text{ kg})}{(3.00 \times 10^8 \text{ m/s})^2} \]

\[ R_s = 2.93 \times 10^{11} \text{ m} = 1.97 \text{ AU} \]

6b.

\[ \frac{1.97 \text{ AU}}{0.080 \text{ AU}} = 24.4 \]